



# Recap on References

# Syntax



We added to  $\lambda_{\rightarrow}$  (with **Unit**) syntactic forms for creating, dereferencing, and assigning reference cells, plus a new type constructor **Ref**.

$t ::=$

`unit`

`x`

`$\lambda x:T.t$`

`t t`

`ref t`

`!t`

`t:=t`

`/`

*terms*

*unit constant*

*variable*

*abstraction*

*application*

*reference creation*

*dereference*

*assignment*

*store location*

# Evaluation



Evaluation becomes *a four-place* relation:  $t \mid \mu \rightarrow t' \mid \mu'$

$$\frac{l \notin \text{dom}(\mu)}{\text{ref } v_1 \mid \mu \longrightarrow l \mid (\mu, l \mapsto v_1)} \quad (\text{E-REFV})$$

$$\frac{\mu(l) = v}{!l \mid \mu \longrightarrow v \mid \mu} \quad (\text{E-DEREFLOC})$$

$$l := v_2 \mid \mu \longrightarrow \text{unit} \mid [l \mapsto v_2]\mu \quad (\text{E-ASSIGN})$$

# Typing



Typing becomes *a three-place* relation:  $\Gamma \mid \Sigma \vdash t : T$

$$\frac{\Sigma(l) = T_1}{\Gamma \mid \Sigma \vdash l : \text{Ref } T_1} \quad (\text{T-LOC})$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : T_1}{\Gamma \mid \Sigma \vdash \text{ref } t_1 : \text{Ref } T_1} \quad (\text{T-REF})$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : \text{Ref } T_{11}}{\Gamma \mid \Sigma \vdash !t_1 : T_{11}} \quad (\text{T-DEREF})$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : \text{Ref } T_{11} \quad \Gamma \mid \Sigma \vdash t_2 : T_{11}}{\Gamma \mid \Sigma \vdash t_1 := t_2 : \text{Unit}} \quad (\text{T-ASSIGN})$$

# Preservation



*Theorem:* if

$$\Gamma \mid \Sigma \vdash t : T$$

$$\Gamma \mid \Sigma \vdash \mu$$

$$t \mid \mu \longrightarrow t' \mid \mu$$

then, for **some**  $\Sigma' \supseteq \Sigma$ ,

$$\Gamma \mid \Sigma' \vdash t' : T$$

$$\Gamma \mid \Sigma' \vdash \mu'.$$

# Progress



*Theorem:* Suppose  $t$  is a *closed, well-typed* term (that is,  $\emptyset \mid \Sigma \vdash t : T$  for some  $T$  and  $\Sigma$ ). Then either  $t$  is a value or else, for any store  $\mu$  such that  $\emptyset \mid \Sigma \vdash \mu$ , there is some term  $t'$  and store  $\mu'$  with  $t \mid \mu \rightarrow t' \mid \mu'$ .

# Nontermination via references



There are well-typed terms in this system that are not strongly normalizing. For example:

$$\begin{aligned} t1 &= \lambda r: \text{Ref} (\text{Unit} \rightarrow \text{Unit}). \\ &\quad (r := (\lambda x: \text{Unit}. (! r)x); \\ &\quad (! r) \text{unit}); \\ t2 &= \text{ref} (\lambda x: \text{Unit}. x); \end{aligned}$$

Applying  $t1$  to  $t2$  yields a (well-typed) divergent term.

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$$t1 = \lambda r: \text{Ref} (\text{Unit} \rightarrow \text{Unit}).$$
$$\boxed{(r := (\lambda x: \text{Unit}. (! r)x);$$
$$(! r) \text{unit});$$
$$t2 = \text{ref} (\lambda x: \text{Unit}. x);$$

Applying  $t1$  to  $t2$  yields a (well-typed) divergent term.





# Chapter 14: Exceptions

Why exceptions

Raising exceptions (aborting whole program)

Handling exceptions

Exceptions carrying values



# Exceptions



# Why exceptions?

Real world programming is full of situations where a function needs to signal to its caller that it is unable to perform its task for :

- Division by zero
- Arithmetic overflow
- Array index out of bound
- Lookup key missing
- File could not be opened
- .....



# Why exceptions?

Most programming languages *provide some mechanism* for *interrupting* the normal flow of control in a program to *signal some exceptional condition* ( & the transfer of control flow) .

*Note that* it is always possible to program *without exceptions* :

- instead of raising an exception, return **None**
- instead of returning result *x* normally, return **Some(x)**

But if we want to wrap every function application in a **case** to find out *whether it returned a result or an exception?*

It is much more convenient to *build this mechanism into the language.*

# Why exceptions?

# type ' $\alpha$  list = None | Some of ' $\alpha$

# let head l = match l with

    []       -> None

  | x::\_     -> Some (x);;

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Type inference?

# Why exceptions?

```
# type 'α list = None | Some of 'α
```

```
# let head l = match l with
```

```
    []      -> None
```

```
  | x::_    -> Some (x);;
```

What is the result of type inference?

```
val head: 'α list -> 'α Option = <fun>
```

```
# let head l = match l with
```

```
    []      -> raise Not_found
```

```
  | x::_    -> x;;
```

```
val head: 'α list -> 'α = <fun>
```

# Varieties of non-local control



There are many ways of adding “*non-local control flow*”

- `exit(1)`
- `goto`
- `setjmp/longjmp`
- `raise/try` (or `catch/throw`) in many variations
- `callcc` / continuations
- more esoteric variants (cf. many Scheme papers)

which allow programs to effect *non-local “jumps”* in the flow of control.

Let’s begin with the simplest of these.





# Raising exceptions

(aborting **whole** program)



# An “abort” primitive in $\lambda_{\rightarrow}$

Raising exceptions (but not catching them), which cause the *abort of the whole program*.

Syntactic forms

$t ::= \dots$   
 $\text{error}$

*terms*  
*run-time error*

Evaluation

$\text{error } t_2 \longrightarrow \text{error}$  (E-APPERR1)

$v_1 \text{ error} \longrightarrow \text{error}$  (E-APPERR2)

# Typing



## Typing

$\Gamma \vdash \text{error} : T$

(T-ERROR)

*New syntactic forms*

$t ::= \dots$

**error**

*terms:*

*run-time error*

*New typing rules*

$\Gamma \vdash t : T$

**$\Gamma \vdash \text{error} : T$**

(T-ERROR)

*New evaluation rules*

$t \rightarrow t'$

**error  $t_2 \rightarrow \text{error}$**

(E-APPERR1)

**$v_1 \text{ error} \rightarrow \text{error}$**

(E-APPERR2)



# Typing errors

Note that the *typing rule* for **error** allows us to give it *any* type **T**.

$$\Gamma \vdash \mathbf{error} : T \quad (\text{T-ERROR})$$

What if we had booleans and numbers in the language?

# Typing errors



Note that the typing rule for **error** allows us to give it *any* type **T**.

$$\Gamma \vdash \mathbf{error} : T \quad (\text{T-ERROR})$$

What if we had booleans and numbers in the language?

This means that both

*if  $x > 0$  then 5 else error*

and

*if  $x > 0$  then true else error*

will typecheck.



# Aside: Syntax-directedness

**Note:** this rule

$$\Gamma \vdash \text{error} : T \quad (\text{T-ERROR})$$

has a *problem* from the *point of view of implementation*:  
it is *not syntax directed*.

This will cause the *Uniqueness of Types* theorem to fail.

For purposes of *defining the language and proving its type safety*, this is not a problem — *Uniqueness of Types* is not critical.

Let's think a little about how the rule might be fixed ...

# Aside: Syntax-directed rules



When we say a set of rules is *syntax-directed* we mean two things:

1. There is *exactly one rule* in the set that applies to each syntactic form (in the sense that we can tell *by the syntax of a term* which rule to use.)
  - e.g., to derive a type for  $t_1 t_2$ , we must use **T-App**.
2. We *don't* have to “*guess*” an input (or output) for any rule.
  - e.g., to derive a type for  $t_1 t_2$ , we need to derive a type for  $t_1$  and a type for  $t_2$ .



# An alternative: Ascription

Can't we just *decorate the error keyword* with its *intended type*, as we have done to fix related problems with other constructs?

$$\Gamma \vdash (\text{error } \boxed{\text{as } T}) : T \quad (\text{T-ERROR})$$



# An alternative : Ascription



Can't we just *decorate the error keyword* with its intended type, as we have done to fix related problems with other constructs?

$$\Gamma \vdash (\text{error } \boxed{\text{as } T}) : T \quad (\text{T-ERROR})$$

No, this doesn't work!

e.g. (assuming our language also has *numbers* and *booleans*) :

succ (if (error as Bool) then 3 else 8)  
→ succ (error as Bool)

# Another alternative: Variable type



In a system with *universal polymorphism* (like OCaml), the variability of typing for **error** can be dealt with by *assigning it a variable type* ?

$\Gamma \vdash \text{error} : ' \alpha$  (T-ERROR)

# Another alternative: Variable type



In a system with *universal polymorphism* (like OCaml), the variability of typing for **error** can be dealt with by assigning it a variable type!

$$\Gamma \vdash \text{error} : 'a \quad (\text{T-ERROR})$$

In effect, we are replacing the *uniqueness of typing* property by a *weaker* (but still very useful) property called *most general typing*.

- i.e., although a *term* may have *many* types, we always have a compact way of *representing* the set of all of its possible types.

# Yet another alternative : *minimal type*



Alternatively, in a system with subtyping (which will be discussed in chapter 15) and *a minimal Bot type*, we can give **error** a unique type:

# Yet another alternative : *minimal* type



Alternatively, in a system with subtyping (which will be discussed in chapter 15) and a *minimal* **Bot** type, we *can* give **error** a unique type:

$$\Gamma \vdash \text{error} : \text{Bot} \quad (\text{T-ERROR})$$

## Note :

What we've really done is *just pushed the complexity* of the old error rule *onto the Bot type*!



# For now...

Let's stick with the original rule

$\Gamma \vdash \text{error} : T$  (T-ERROR)

and live with the resulting *non-determinism* of the typing relation.

# Type safety



Property of preservation?

The preservation theorem requires *no changes* when we add **error**:

if a term of type **T** reduces to **error**, that's fine, since **error** has every type **T**.

# Type safety



Property of preservation?

The preservation theorem requires no changes when we add **error** :

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Whereas,

Progress requires a little more care.



# Progress



First, *note that* we do *not* want to extend the set of values to include *error*, since this would make our new rule for *propagating errors* through applications.

$$v_1 \text{ error} \longrightarrow \text{error} \quad (\text{E-APPERR2})$$

*overlap with* our existing computation rule for applications:

$$(\lambda x:T_{11}. t_{12}) v_2 \longrightarrow [x \mapsto v_2]t_{12} \quad (\text{E-APPABS})$$

e.g, the term

$$(\lambda x:\text{Nat}. 0) \text{ error}$$

could evaluate to either **0** (which would be wrong) or **error** (which is what we intend).

# Progress



Instead, we keep **error** as a *non-value normal form*, and refine the statement of progress to explicitly mention the *possibility* that *terms may evaluate to error* instead of to a value.

Theorem [Progress]:     *Suppose  $t$  is a closed, well-typed normal form. Then either  $t$  is a value or  $t = \text{error}$ .*



# Handling exceptions

# Catching exceptions

$t ::= \dots$  *terms*  
 $\text{try } t \text{ with } t$  *trap errors*

## Evaluation

$\text{try } v_1 \text{ with } t_2 \longrightarrow v_1$  (E-TRYV)

$\text{try error with } t_2 \longrightarrow t_2$  (E-TRYERROR)

$$\frac{t_1 \longrightarrow t'_1}{\text{try } t_1 \text{ with } t_2 \longrightarrow \text{try } t'_1 \text{ with } t_2} \quad (\text{E-TRY})$$

## Typing

$$\frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : T}{\Gamma \vdash \text{try } t_1 \text{ with } t_2 : T} \quad (\text{T-TRY})$$



# Exceptions carrying values



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When something unusual happened, it's useful to *send back some extra information* about *which unusual thing has happened* so that the handler can *take some actions* depending on this information.



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`t ::= ...`  
`raise t`

*terms*  
*raise exception*



# Exceptions carrying values

When something unusual happened, it's useful to *send back some extra information* about *which unusual thing has happened* so that the handler can *take some actions* depending on this information.

$t ::= \dots$   
`raise t`

terms  
*raise exception*

Atomic term `error` is replaced by a *term constructor*  
`raise t`

where  $t$  is the *extra information* that we want to *pass to the exception handler*.



# Evaluation



$(\text{raise } v_{11}) t_2 \longrightarrow \text{raise } v_{11}$  (E-APPRAISE1)

$v_1 (\text{raise } v_{21}) \longrightarrow \text{raise } v_{21}$  (E-APPRAISE2)

$$\frac{t_1 \longrightarrow t'_1}{\text{raise } t_1 \longrightarrow \text{raise } t'_1} \quad (\text{E-RAISE})$$

$\text{raise } (\text{raise } v_{11}) \longrightarrow \text{raise } v_{11}$  (E-RAISERAISE)

$\text{try } v_1 \text{ with } t_2 \longrightarrow v_1$  (E-TRYV)

$\text{try } \text{raise } v_{11} \text{ with } t_2 \longrightarrow t_2 v_{11}$  (E-TRYRAISE)

$$\frac{t_1 \longrightarrow t'_1}{\text{try } t_1 \text{ with } t_2 \longrightarrow \text{try } t'_1 \text{ with } t_2} \quad (\text{E-TRY})$$

# Evaluation



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$$\frac{t_1 \longrightarrow t'_1}{\text{try } t_1 \text{ with } t_2 \longrightarrow \text{try } t'_1 \text{ with } t_2}$$
 (E-TRY)

# Typing



To typecheck **raise** expressions, we need to *choose a type for the values* that are carried along with exceptions, let's call it  $T_{\text{exn}}$

$$\frac{\Gamma \vdash t_1 : T_{\text{exn}}}{\Gamma \vdash \text{raise } t_1 : T} \quad (\text{T-RAISE})$$

$$\frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : T_{\text{exn}} \rightarrow T}{\Gamma \vdash \text{try } t_1 \text{ with } t_2 : T} \quad (\text{T-TRY})$$



# What is $T_{exn}$ ?

Further, we need to decide *what type* to use as  $T_{exn}$ . There are *several possibilities*.

1. Numeric error codes:  $T_{exn} = \text{Nat}$  (as in Unix)
2. Error messages:  $T_{exn} = \text{String}$
3. A *predefined* variant type:

```
 $T_{exn}$  = <divideByZero:    Unit,  
          overflow:      Unit,  
          fileNotFound:  String,  
          fileNotReadable: String,  
          ... >
```

4. An *extensible* variant type (as in Ocaml)
5. A *class* of “*throwable objects*” (as in Java)

# Recapitulation: Error handling



→ error **try**

Extends  $\lambda_{\rightarrow}$  with errors (14-1)

*New syntactic forms*

$t ::= \dots$   
**try  $t$  with  $t$**

*terms:*  
*trap errors*

$$\frac{t_1 \rightarrow t'_1}{\text{try } t_1 \text{ with } t_2 \rightarrow \text{try } t'_1 \text{ with } t_2}$$
 (E-TRY)

*New evaluation rules*

**try  $v_1$  with  $t_2 \rightarrow v_1$**

$t \rightarrow t'$   
(E-TRYV)

*New typing rules*

$\Gamma \vdash t : T$   
(T-TRY)

**try error with  $t_2$   
 $\rightarrow t_2$**

(E-TRYERROR)

$$\frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : T}{\Gamma \vdash \text{try } t_1 \text{ with } t_2 : T}$$

# Recapitulation: Exceptions carrying values

→ *exceptions*

*Extends  $\lambda_{\dots}$  (9-1)*

*New syntactic forms*

$t ::= \dots$

$\text{raise } t$

$\text{try } t \text{ with } t$

*terms:*

*raise exception  
handle exceptions*

*New evaluation rules*

$t \rightarrow t'$

$(\text{raise } v_{11}) t_2 \rightarrow \text{raise } v_{11}$  (E-APPRAISE1)

$v_1 (\text{raise } v_{21}) \rightarrow \text{raise } v_{21}$  (E-APPRAISE2)

$\frac{t_1 \rightarrow t'_1}{\text{raise } t_1 \rightarrow \text{raise } t'_1}$  (E-RAISE)

$\text{raise } (\text{raise } v_{11}) \rightarrow \text{raise } v_{11}$  (E-RAISERAISE)

$\text{try } v_1 \text{ with } t_2 \rightarrow v_1$  (E-TRYV)

$\text{try raise } v_{11} \text{ with } t_2 \rightarrow t_2 v_{11}$  (E-TRYRAISE)

$\frac{t_1 \rightarrow t'_1}{\text{try } t_1 \text{ with } t_2 \rightarrow \text{try } t'_1 \text{ with } t_2}$  (E-TRY)

*New typing rules*

$\Gamma \vdash t : T$

$\frac{\Gamma \vdash t_1 : T_{\text{exn}}}{\Gamma \vdash \text{raise } t_1 : T}$  (T-EXN)

$\frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : T_{\text{exn}} \rightarrow T}{\Gamma \vdash \text{try } t_1 \text{ with } t_2 : T}$  (T-TRY)

# Recapitulation



Raising exception is *more than an error mechanism*: it's a *programmable control structure*

- Sometimes a way to quickly *escape from the computation*.
- And allow programs to effect *non-local “jumps”* in the flow of control by setting a *handler* during evaluation of an expression that may be invoked by raising an exception.
- Exceptions are *value-carrying* in the sense that one may pass a value to *the exception handler* when the exception is raised.
- Exception values have a single type,  $T_{exn}$ , which is *shared by all exception handler*.

# Recapitulation



E.g., Exceptions are used in **OCaml** as a *control mechanism*, **either** to signal errors, **or** to control the flow of execution.

- When an exception is raised, the current execution is aborted, and control is thrown to the most recently entered active exception handler, which may choose to handle the exception, or pass it through to the next exception handler.
- $T_{exn}$  is defined to be an extensible data type, in the sense that new constructors may be introduced using exception declaration, with no restriction on the types of value that may be associated with the constructor.





# Examples in OCaml

```
# let rec assoc key = function
  (k, v) :: l ->
    if k = key then v
    else assoc key l
  | [] -> raise Not_found;;
val assoc : 'a -> ('a * 'b) list -> 'b = <fun>
```

```
# assoc 2 l;;
- : string = "World"
# assoc 3 l;;
- Exception: Not_found.
# "Hello" ^ assoc 2 l;;
- : string = "HelloWorld"
```

# Examples in OCaml



```
let find_index p =  
  let rec find n =  
    function [] -> raise (Failure "not found")  
      | x::L -> if p(x) then raise (Found n)  
                else find (n+1) L  
  in  
  try find 1 L with Found n -> n;;
```

# HW for chap14

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- 14.3.1