Recap on References

Syntax

We added to $\lambda_{\to}$ (with Unit) syntactic forms for creating, dereferencing, and assigning reference cells, plus a new type constructor Ref.

t ::=		*terms*
	unit	*unit constant*
	x	*variable*
	λx:T.t	*abstraction*
	t t	*application*
	ref t	*reference creation*
	!t	*dereference*
	t:=t	*assignment*
	l	store location

Evaluation

Evaluation becomes *a four-place* relation: $t \mid \mu \longrightarrow t' \mid \mu'$

$$\frac{l \notin dom(\mu)}{\texttt{ref}\ v_1 \mid \mu \longrightarrow l \mid (\mu, l \mapsto v_1)} \qquad \text{(E-RefV)}$$

$$\frac{\mu(l) = v}{!l \mid \mu \longrightarrow v \mid \mu} \qquad \text{(E-DerefLoc)}$$

$$l := v_2 \mid \mu \longrightarrow \texttt{unit} \mid [l \mapsto v_2]\mu \qquad \text{(E-Assign)}$$

Typing

Typing becomes *a three-place* relation: $\Gamma \mid \Sigma \vdash t : T$

$$\frac{\Sigma(l) = T_1}{\Gamma \mid \Sigma \vdash l : \text{Ref } T_1} \quad \text{(T-Loc)}$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : T_1}{\Gamma \mid \Sigma \vdash \text{ref } t_1 : \text{Ref } T_1} \quad \text{(T-Ref)}$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : \text{Ref } T_{11}}{\Gamma \mid \Sigma \vdash !t_1 : T_{11}} \quad \text{(T-Deref)}$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : \text{Ref } T_{11} \qquad \Gamma \mid \Sigma \vdash t_2 : T_{11}}{\Gamma \mid \Sigma \vdash t_1 := t_2 : \text{Unit}} \quad \text{(T-Assign)}$$

Preservation

Theorem: if

$$\Gamma \mid \Sigma \vdash t : T$$

$$\Gamma \mid \Sigma \vdash \mu$$

$$t \mid \mu \longrightarrow t' \mid \mu$$

then, for some $\Sigma' \supseteq \Sigma$,

$$\Gamma \mid \Sigma' \vdash t' : T$$

$$\Gamma \mid \Sigma' \vdash \mu'.$$

Progress

Theorem: Suppose t is a *closed, well-typed* term (that is, $\emptyset \mid \Sigma \vdash t : T$ for some T and Σ). Then either t is a value or else, for any store μ such that $\emptyset \mid \Sigma \vdash \mu$, there is some term t' and store μ' with $t \mid \mu \longrightarrow t' \mid \mu'$.

Chapter 14: Exceptions

Why exceptions

Raising exceptions (aborting whole program)

Handling exceptions

Exceptions carrying values


PEKING UNIVERSITY
1898

Exceptions


NII
国立情報学研究所
National Institute of Informatics

Why exceptions?

Real world programming is full of situations where a function needs to *signal to it caller* that it is *unable to perform its task* for :

- Division by zero
- Arithmetic overflow
- Array index out of bound
- Lookup key missing
- File could not be opened
- ……

Why exceptions?

Most programming languages *provide some mechanism* for interrupting *the normal flow of control* in a program to *signal some exceptional condition* (& the transfer of control flow) .

Note that it is always possible to program *without exceptions* :

- instead of raising an exception, return None
- instead of returning result x normally, return Some(x)

But if we want to wrap every function application in a case to find out *whether it returned a result or an exception*?

It is much more convenient to *build this mechanism into the language.*

Why exceptions?


```
# type 'α list = None | Some of 'α

# let head l = match l with
                 []     -> None
               | x::_   -> Some (x);;
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Type inference?

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# type 'α list = None | Some of 'α
# let head l  = match l with
                    []      -> None
                |   x::_    -> Some (x);;
```

What is the result of type inference?

```
val head: 'α list -> 'α Option = <fun>
# let head l   = match l with
                    []      -> raise Not_found
                |   x::_    -> x;;
val head: 'α list -> 'α = <fun>
```

Varieties of non-local control

There are many ways of adding "*non-local control flow*"

- exit(1)
- goto
- setjmp/longjmp
- raise/try (or catch/throw) in many variations
- callcc / continuations
- more esoteric variants (cf. many Scheme papers)

which allow programs to effect *non-local "jumps"* in the flow of control.

Let's begin with the simplest of these.

Raising exceptions

(aborting whole program)

An "abort" primitive in $\lambda_{\to}$

Raising exceptions (but not catching them), which cause the *abort of the whole program*.

Syntactic forms

t	::=	...	*terms*
		error	*run-time error*

Evaluation

$$\texttt{error}\ t_2 \longrightarrow \texttt{error} \qquad \text{(E-AppErr1)}$$

$$v_1\ \texttt{error} \longrightarrow \texttt{error} \qquad \text{(E-AppErr2)}$$

Typing

Typing

$$\Gamma \vdash \texttt{error} : T \qquad \text{(T-Error)}$$

New syntactic forms

$t ::= \ldots$ *terms:*

$\texttt{error}$ *run-time error*

New evaluation rules $t \longrightarrow t'$

$\texttt{error}\ t_2 \longrightarrow \texttt{error}$ (E-AppErr1)

$v_1\ \texttt{error} \longrightarrow \texttt{error}$ (E-AppErr2)

New typing rules $\Gamma \vdash t : T$

$\Gamma \vdash \texttt{error} : T$ (T-Error)

Typing errors

Note that the *typing rule* for error allows us to give it *any* type T.

$$\Gamma \vdash \texttt{error} : \texttt{T} \qquad \text{(T-Error)}$$

What if we had booleans and numbers in the language?

Typing errors

Note that the typing rule for error allows us to give it *any* type T.

$$\Gamma \vdash \texttt{error} : \texttt{T} \qquad \text{(T-Error)}$$

What if we had booleans and numbers in the language?

This means that both

$$if\ x > 0\ then\ 5\ else\ error$$

and

$$if\ x > 0\ then\ true\ else\ error$$

will typecheck.

Aside: Syntax-directedness

Note: this rule

$$\Gamma \vdash \texttt{error} : \texttt{T} \qquad \text{(T-Error)}$$

has a *problem* from the *point of view of implementation*: it is *not syntax directed*.

This will cause the *Uniqueness of Types* theorem to fail.

For purposes of *defining the language and proving its type safety*, this is not a problem — *Uniqueness of Types* is not critical.

Let's think a little about how the rule might be fixed ...

Aside: Syntax-directed rules

When we say a set of rules is *syntax-directed* we mean two things:

1. There is *exactly one rule* in the set that applies to each syntactic form (in the sense that we can tell *by the syntax of a term* which rule to use.)
   - *e.g.,* to derive a type for $t_1\ t_2$, we must use T-App.
2. We *don't* have to "*guess*" an input (or output) for any rule.
   - *e.g.,* to derive a type for $t_1\ t_2$, we need to derive a type for t_1 and a type for t_2.

An alternative: Ascription

Can't we just *decorate the* error *keyword* with its *intended type*, as we have done to fix related problems with other constructs?

$$\Gamma \vdash (\texttt{error as T}) : \texttt{T} \qquad \text{(T-Error)}$$

An alternative : Ascription

Can't we just *decorate the error keyword* with its intended type, as we have done to fix related problems with other constructs?

$$\Gamma \vdash (\texttt{error as T}) : \texttt{T} \qquad \text{(T-Error)}$$

No, this doesn't work!

e.g. assuming our language also has *numbers* and *booleans*:

succ (if (error as Bool) then 3 else 8)

$\longrightarrow$ succ (error as Bool)

Another alternative: Variable type

In a system with *universal polymorphism* (like OCaml), the variability of typing for error can be dealt with by *assigning it a variable type* ?

$$\Gamma \vdash \text{error} : {'\alpha} \qquad \text{(T-ERROR)}$$

Another alternative: Variable type

In a system with *universal polymorphism* (like OCaml), the variability of typing for error can be dealt with by assigning it a variable type!

$$\Gamma \vdash \text{error} : {}'\alpha \qquad \text{(T-ERROR)}$$

In effect, we are replacing the *uniqueness of typing* property by a *weaker* (but still very useful) property called *most general typing*.

- i.e., although a *term* may have *many types*, we always have a compact way of *representing* the set of all of its possible types.

Yet another alternative : *minimal* type

Alternatively, in a system with subtyping (which will be discussed in chapter 15) and *a minimal* Bot type, we *can* give error a unique type:

Yet another alternative : *minimal* type

Alternatively, in a system with subtyping (which will be discussed in chapter 15) and *a minimal* Bot type, we *can* give error a unique type:

$$\Gamma \vdash \texttt{error} : \texttt{Bot} \qquad \text{(T-Error)}$$

Note :

What we've really done is *just pushed the complexity* of the old error rule *onto the* Bot *type*!

For now...

Let's stick with the original rule

$$\Gamma \vdash \texttt{error} : \texttt{T} \qquad \text{(T-Error)}$$

and live with the resulting *non-determinism* of the typing relation.

Type safety

Property of preservation?

The preservation theorem requires *no changes* when we add error:

> if a term of type T reduces to error, that's fine, since error has every type T.

Type safety

Property of preservation?

The preservation theorem requires no changes when we add error: :

> if a term of type T reduces to error, that's fine, since error has every type T.

Whereas,

Progress requires a little more care.

Progress

First, *note that* we do *not* want to extend the set of values to include error, since this would make our new rule for *propagating errors* through applications.

$$v_1 \ \texttt{error} \longrightarrow \texttt{error} \qquad \text{(E-AppErr2)}$$

overlap with our existing computation rule for applications:

$$(\lambda x{:}T_{11}.t_{12}) \ v_2 \longrightarrow [x \mapsto v_2]t_{12} \quad \text{(E-AppAbs)}$$

e.g, the term

$$(\lambda\, x{:}\,\text{Nat}.\,0)\ \texttt{error}$$

could evaluate to either 0 (which would be wrong) or error (which is what we intend).

Progress

Instead, we keep error as a *non-value normal form*, and **refine the statement of progress** to explicitly mention the *possibility* that *terms may evaluate to error* instead of to a value.

Theorem [Progress]: *Suppose* t *is a closed, well-typed normal form. Then either* t *is a value or* $t = error$.

Handling exceptions

Catching exceptions

$$t ::= \ldots \qquad \textit{terms}$$
$$\texttt{try } t \texttt{ with } t \qquad \textit{trap errors}$$

Evaluation

$$\texttt{try } v_1 \texttt{ with } t_2 \longrightarrow v_1 \qquad \text{(E-TryV)}$$

$$\texttt{try error with } t_2 \longrightarrow t_2 \qquad \text{(E-TryError)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{try } t_1 \texttt{ with } t_2 \longrightarrow \texttt{try } t_1' \texttt{ with } t_2} \qquad \text{(E-Try)}$$

Typing

$$\frac{\Gamma \vdash t_1 : T \qquad \Gamma \vdash t_2 : T}{\Gamma \vdash \texttt{try } t_1 \texttt{ with } t_2 : T} \qquad \text{(T-Try)}$$

Exceptions carrying values

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When something unusual happened, it's useful to *send back some extra information* about *which unusual thing has happened* so that the handler can *take some actions* depending on this information.

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Exceptions carrying values

When something unusual happened, it's useful to *send back some extra information* about *which unusual thing has happened* so that the handler can *take some actions* depending on this information.

t ::=	...	*terms*
	raise t	*raise exception*

Atomic term error is replaced by *a term constructor*

raise t

where *t* is the *extra information* that we want to *pass to the exception handler*.

Evaluation

$$(\texttt{raise}\ v_{11})\ t_2 \longrightarrow \texttt{raise}\ v_{11} \qquad \text{(E-AppRaise1)}$$

$$v_1\ (\texttt{raise}\ v_{21}) \longrightarrow \texttt{raise}\ v_{21} \qquad \text{(E-AppRaise2)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{raise}\ t_1 \longrightarrow \texttt{raise}\ t_1'} \qquad \text{(E-Raise)}$$

$$\texttt{raise}\ (\texttt{raise}\ v_{11}) \longrightarrow \texttt{raise}\ v_{11} \qquad \text{(E-RaiseRaise)}$$

$$\texttt{try}\ v_1\ \texttt{with}\ t_2 \longrightarrow v_1 \qquad \text{(E-TryV)}$$

$$\texttt{try raise}\ v_{11}\ \texttt{with}\ t_2 \longrightarrow t_2\ v_{11} \qquad \text{(E-TryRaise)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{try}\ t_1\ \texttt{with}\ t_2 \longrightarrow \texttt{try}\ t_1'\ \texttt{with}\ t_2} \qquad \text{(E-Try)}$$

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$$(\texttt{raise}\ v_{11})\ t_2 \longrightarrow \texttt{raise}\ v_{11} \qquad \text{(E-AppRaise1)}$$

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Typing

To typecheck raise expressions, we need to *choose a type for the values* that are carried along with exceptions, let's call it T_{exn}

$$\frac{\Gamma \vdash t_1 : T_{exn}}{\Gamma \vdash \text{raise } t_1 : T} \qquad \text{(T-RAISE)}$$

$$\frac{\Gamma \vdash t_1 : T \qquad \Gamma \vdash t_2 : T_{exn} \rightarrow T}{\Gamma \vdash \text{try } t_1 \text{ with } t_2 : T} \qquad \text{(T-TRY)}$$

What is T_{exn} ?

Further, we need to decide *what type* to use as T_{exn}. There are *several possibilities*.

1. Numeric error codes: T_{exn} = Nat (as in Unix)
2. Error messages: T_{exn} = String
3. A *predefined* variant type:

```
T_exn  =  <divideByZero:     Unit,
           overflow:         Unit,
           fileNotFound:     String,
           fileNotReadable:  String,
           ... >
```

4. An *extensible* variant type (as in Ocaml)
5. A *class* of *"throwable objects"* (as in Java)

Recapitulation： Error handling

→ *error* *try*

Extends $\lambda_{\to}$ *with errors (14-1)*

New syntactic forms

$t ::= \ldots$

$\texttt{try } t \texttt{ with } t$

terms: trap errors

$t \longrightarrow t'$

New evaluation rules

$$\texttt{try } v_1 \texttt{ with } t_2 \longrightarrow v_1 \quad \text{(E-TryV)}$$

$$\texttt{try error with } t_2 \longrightarrow t_2 \quad \text{(E-TryError)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{try } t_1 \texttt{ with } t_2 \longrightarrow \texttt{try } t_1' \texttt{ with } t_2} \quad \text{(E-Try)}$$

New typing rules

$\Gamma \vdash t : T$

$$\frac{\Gamma \vdash t_1 : T \qquad \Gamma \vdash t_2 : T}{\Gamma \vdash \texttt{try } t_1 \texttt{ with } t_2 : T} \quad \text{(T-Try)}$$

Recapitulation： Exceptions carrying values

→ *exceptions* — *Extends $\lambda_{\to}$ (9-1)*

New syntactic forms

$t ::= \ldots$ — *terms:*

$\texttt{raise}\ t$ — *raise exception*

$\texttt{try}\ t\ \texttt{with}\ t$ — *handle exceptions*

New evaluation rules $\boxed{t \longrightarrow t'}$

$$(\texttt{raise}\ v_{11})\ t_2 \longrightarrow \texttt{raise}\ v_{11} \quad \text{(E-AppRaise1)}$$

$$v_1\ (\texttt{raise}\ v_{21}) \longrightarrow \texttt{raise}\ v_{21} \quad \text{(E-AppRaise2)}$$

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$$\texttt{raise}\ (\texttt{raise}\ v_{11}) \longrightarrow \texttt{raise}\ v_{11} \quad \text{(E-RaiseRaise)}$$

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New typing rules $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_{exn}}{\Gamma \vdash \texttt{raise}\ t_1 : T} \quad \text{(T-Exn)}$$

$$\frac{\Gamma \vdash t_1 : T \qquad \Gamma \vdash t_2 : T_{exn} \to T}{\Gamma \vdash \texttt{try}\ t_1\ \texttt{with}\ t_2 : T} \quad \text{(T-Try)}$$

Recapitulation

Raising exception is *more than an error mechanism*: it's a *programmable control structure*

- Sometimes a way to quickly *escape from the computation.*
- And allow programs to effect *non-local "jumps"* in the flow of control by setting a *handler* during evaluation of an expression that may be invoked by raising an exception.
- Exceptions are *value-carrying* in the sense that one may pass a value to *the exception handler* when the exception is raised.
- Exception values have a single type, T_{exn} , which is *shared by all exception handler*.

Recapitulation

E.g., Exceptions are used in OCaml as a *control mechanism*, either to signal errors, or to control the flow of execution.

- When an exception is raised, the current execution is aborted, and control is thrown to the most recently entered active exception handler, which may choose to handle the exception, or pass it through to the next exception handler.
- T_{exn} is defined to be an extensible data type, in the sense that new constructors may be introduced using exception declaration, with no restriction on the types of value that may be associated with the constructor.

Examples in OCaml

```
# let rec assoc key = function
    (k, v) :: l ->
          if k = key then v
          else assoc key l
    | [] -> raise Not_found;;
val assoc : 'a -> ('a * 'b) list -> 'b = <fun>

# assoc 2 l;;
-   : string = "World“
# assoc 3 l;;
-   Exception: Not_found.
# "Hello" ^ assoc 2 l;;
- : string = "HelloWorld"
```

Examples in OCaml

```
let find_index p =
    let rec find n =
        function  []     -> raise (Failure "not found")
                | x::L ->  if p(x) then raise (Found n)
                                   else find (n+1) L
    in
        try find 1 L  with Found n -> n;;
```

HW for chap14

- 14.3.1